

$$x^2y + 3xy^3 - x = 3$$

$$\begin{array}{ll} f(x) = x^2 & g(x) = y \\ f'(x) = 2x & g'(x) = \frac{dy}{dx} \end{array}$$

$$\begin{array}{ll} f(x) = 3x & g(x) = y^3 \\ f'(x) = 3 & g'(x) = 3y^2 \frac{dy}{dx} \end{array}$$

$$\left(x^2 \cdot \frac{dy}{dx}\right) + (2x \cdot y) + \left(3x \cdot 3y^2 \frac{dy}{dx}\right) + 3y^3 - 1 = 0$$

$$\left(x^2 \frac{dy}{dx} + 9xy^2 \frac{dy}{dx}\right) + (2xy + 3y^3) = 1$$

$$\frac{dy}{dx} (x^2 + 9xy^2) = 1 - (2xy + 3y^3)$$

$$\boxed{\frac{dy}{dx} = \frac{1 - (2xy + 3y^3)}{x^2 + 9xy^2}}$$

$$xy^3 = (1 + \sec y)(1 + y^4)$$

$$\frac{xy^3}{1 + \sec y} = \frac{1 + y^4}{1}$$

$$\frac{d}{dx} \left( \frac{xy^3}{1 + \sec y} \right) = \frac{\frac{d}{dx} (xy^3)(1 + \sec y) - (xy^3) \left( \frac{d}{dx} (1 + \sec y) \right)}{(1 + \sec y)^2}$$

$$\frac{d}{dx} \left( \frac{xy^3}{1 + \sec y} \right) = \frac{(1 \cdot y^3 + x \cdot 3y^2 \frac{dy}{dx})(1 + \sec y) - (xy^3)(\sec y \tan y \frac{dy}{dx})}{(1 + \sec y)^2}$$

$$\frac{dy}{dx} = \frac{y^3(y + x \cdot 3 \frac{dy}{dx})(1 + \sec y) - (xy^3)(\sec y \tan y \frac{dy}{dx})}{4y^3(1 + \sec y)^2}$$

$$\frac{dy}{dx} = \frac{(y^2)(1 + \sec y)(y + 3x \frac{dy}{dx})}{4y^3(1 + \sec y)^2} - \frac{xy^3 \sec y \tan y \frac{dy}{dx}}{4y^3(1 + \sec y)^2}$$

$$\frac{dy}{dx} = \frac{y + 3x \frac{dy}{dx}}{4y(1 + \sec y)} - \frac{x \sec y \tan y \frac{dy}{dx}}{4(1 + \sec y)^2}$$

$$\frac{dy}{dx} = \frac{(y)}{4y(1 + \sec y)} + \frac{3x \frac{dy}{dx}}{4y(1 + \sec y)} - \frac{x \sec y \tan y \frac{dy}{dx}}{4(1 + \sec y)^2}$$

$$\frac{dy}{dx} - \frac{3x}{4y(1 + \sec y)} \frac{dy}{dx} + \frac{x \sec y \tan y \frac{dy}{dx}}{4(1 + \sec y)^2} = \frac{1}{4(1 + \sec y)}$$

$$\frac{dy}{dx} \left( 1 - \frac{3x}{4y(1 + \sec y)} + \frac{x \sec y \tan y}{4(1 + \sec y)^2} \right) = \frac{1}{4(1 + \sec y)}$$

$$xy^3 = (1 + \sec y)(1 + y^4)$$

$$(y^3 + 3xy^2 \frac{dy}{dx}) = (\sec y \tan y \frac{dy}{dx})(1 + y^4) + (1 + \sec y)(4y^3 \frac{dy}{dx})$$

$$3xy^2 \frac{dy}{dx} - \sec y \tan y (1 + y^4) \frac{dy}{dx} - 4y^3 (1 + \sec y) \frac{dy}{dx} = -y^3$$

$$\frac{dy}{dx} (3xy^2 - \sec y \tan y (1 + y^4) - 4y^3 (1 + \sec y)) = -y^3$$

$$\frac{dy}{dx} = \frac{-y^3}{3xy^2 - \sec y \tan y (1 + y^4) - 4y^3 (1 + \sec y)}$$

$$\underline{3.6/14} \quad x^3 y^2 - 5x^2 y + x = 1$$

$$\left[ \frac{d}{dx}(x^3) \cdot y^2 + x^3 \cdot \frac{d}{dx}(y^2) \right] - 5 \left[ \frac{d}{dx}(x^2) \cdot y + x^2 \frac{dy}{dx} \right] + 1 = 0$$

$$3x^2 y^2 + 2yx^3 \frac{dy}{dx} - 5(2xy + x^2 \frac{dy}{dx}) = -1$$

$$2x^3 y \frac{dy}{dx} - 5x^2 \frac{dy}{dx} = -1 - 3x^2 y^2 + 10xy$$

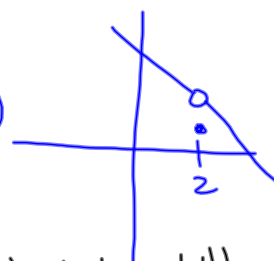
$$\frac{dy}{dx} = \frac{-1 - 3x^2 y^2 + 10xy}{2x^3 y - 5x^2} = \frac{-(-1 - 3x^2 y^2 + 10xy)}{-(2x^3 y - 5x^2)}$$

$$= \frac{1 + 3x^2 y^2 - 10xy}{-2x^3 y + 5x^2}$$

## chapter 2

### 2.1) Limits intuitively

'everyone' missed  $\lim_{x \rightarrow 2} f(x)$



### 2.2) Interior limits

$$\frac{0}{0}, \frac{\infty}{\infty} \quad \text{not } \frac{0}{0} < \begin{matrix} +\infty \\ -\infty \end{matrix}$$

- relationship to vertical asymp

valuable skill  
think like an  
x getting close  
to a # but not  
hitting it

### 2.3) limits to infinity

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty$$

- relationship to horizontal asymp

= techniques for 2.2 and 2.3

factor / cancel

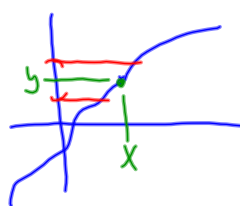
multiply by conjugate  $\rightarrow$  cancel

create a fraction  $\rightarrow$  cancel

factor out powers of x  $\rightarrow$

$$\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2}}{x}$$

2.4



$$y + \epsilon = f(x + \delta)$$

$$y - \epsilon = f(x - \delta)$$

### 2.5) continuity: 3 prong def

find K that makes ..... continuous

### 2.6) trig limits

in particular

$$\lim_{x \rightarrow 0} \frac{\sin x}{x}, \lim_{x \rightarrow 0} \frac{1 - \cos x}{x}$$